DISCRETE DISTRIBUTIONS

Binomial distribution

A **binomial distribution** can be thought of as simply the probability of a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times. The binomial is a type of distribution that has **two possible outcomes**. For example, a coin toss has only two possible outcomes: heads or tails and taking a test could have two possible outcomes: pass or fail.

Binomial distributions must also meet the following three criteria:

1. **The number of observations or trials is fixed.** In other words, you can only figure out the probability of something happening if you do it a certain number of times. This is common sense—if you toss a coin once, your probability of getting a tail is 50%. If you toss a coin 20 times, your probability of getting tails is very, very close to 100%.
2. **Each observation or trial is independent.** In other words, none of your trials have an effect on the probability of the next trial.
3. The **probability of success** (tails, heads, fail or pass) is **exactly the same** from one trial to another.

The probability mass function of Binomial distribution is given by

$$P(X) = \frac{n!}{(n-X)!\,X!} \cdot (p)^X \cdot (q)^{n-X}$$

EXAMPLE

80% of people who purchase pet insurance are women. If 9 pet insurance owners are randomly selected, find the probability that exactly 6 are women.

Step 1: Identify 'n' from the problem. Using our sample question, n (the number of randomly selected items) is 9.
Step 2: Identify 'X' from the problem. X (the number you are asked to find the probability for) is 6.
Step 3: Work the first part of the formula. The first part of the formula is
n! / (n – X)! X!
Substitute your variables:
9! / ((9 – 6)! × 6!)
Which equals 84. Set this number aside for a moment.
Step 4: Find p and q. p is the probability of success and q is the probability of failure. We are given p = 80%, or .8. So the probability of failure is 1 – .8 = .2 (20%).
Step 5: Work the second part of the formula.
$p^X = .8^6$
= .262144
Set this number aside for a moment.
Step 6: Work the third part of the formula.
$q^{(n-X)} = .2^{(9-6)}$
$= .2^3$
= .008
Step 7: Multiply your answer from step 3, 5, and 6 together.
84 × .262144 × .008 = 0.176.
The probability exactly 6 are women is 0.176.

Geometric Distribution

The geometric distribution represents the number of failures before you get a success in a series of Bernoulli trials. This discrete probability distribution is represented by the probability density function:

$f(x) = (1 - p)^{x-1} * p$

EXAMPLE: If your probability of success is 0.2, what is the probability you meet an independent voter on your third try?

Inserting 0.2 as p and with X = 3, the probability density function becomes:

$f(x) = (1 - p)^{x-1} * p$

$P(X=3) = (1 - 0.2)^{3-1}(0.2)$

$P(X=3) = (0.8)^2 * 0.2 = 0.128$.

Poisson Distribution

A Poisson distribution is a tool that helps to predict the probability of certain events from happening when you know how often the event has occurred. It gives us the **probability of a given number of events happening in a fixed interval of time**.

The Poisson Distribution pmf is: $P(x; \mu) = (e^{-\mu} * \mu^x) / x!$

EXAMPLE: The average number of major storms in your city is 2 per year. What is the probability that exactly 3 storms will hit your city next year?

Step 1: Figure out the components you need to put into the equation.

- $\mu = 2$ (average number of storms per year, historically)
- $x = 3$ (the number of storms we think might hit next year)
- $e = 2.71828$ (e is Euler's number, a constant)

Step 2: Plug the values from Step 1 into the Poisson distribution formula:

- $P(x; \mu) = (e^{-\mu}) (\mu^x) / x!$
- $= (2.71828^{-2}) (2^3) / 3!$
- $= (0.13534) (8) / 6$
- $= 0.180$

The probability of 3 storms happening next year is 0.180, or 18%

Practice questions

1. **60% of people who purchase sports cars are men. If 10 sports car owners are randomly selected, find the probability that exactly 7 are men.**
2. **A coin is tossed 10 times. What is the probability of getting exactly 6 heads?**
3. **Prove that sum of the independent poisson variate is a poissan variate.**
4. **Obtain the moment generating function of the geometric distribution.**