Baryon Wavefunction

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Anti-symmetric Total Wavefunction

Total wavefunction fo r baryons is

$$\Psi_{\textbf{baryon}} = \Psi_{\textbf{quark flavor}} \Psi_{\textbf{quark space}} \Psi_{\textbf{quark spin}} \Psi_{\textbf{quark color}}$$

$\Psi_{\text{quark color}} \longrightarrow \frac{1}{\sqrt{6}}[RGB + BRG + GBR - RBG - BGR - GRB]$ **singlet and anti-symmetric (for baryons)**

Also for ground state , l = 0 ; $\Psi_{\text{quark space}} \longrightarrow$ symmetric

Thus , for $\Psi_{\textbf{baryon}}$ to be **anti-symmetric** ; combination of $\Psi_{\textbf{quark flavor}}$ & $\Psi_{\textbf{quark spin}}$ should be **symmetric**

Spin Combination

- 8 Possible spin states are (↑↑↑), (↑↑↓), (↑↓↓), (↓↑↑), (↓↑↓), (↓↓↑), (↑↓↑), (↓↓↓)
- Quark spin combination gives (i) **spin 3/2 decuplets (Ψ_s)** **(ii) spin ½ octets [(Ψ_{12}) or (Ψ_{23})]**

DECUPLETS

$$\left|\begin{matrix} spin & projection \\ \frac{3}{2} & \frac{3}{2} \end{matrix}\right\rangle = (\uparrow\uparrow\uparrow)$$

$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)/\sqrt{3}$$

$$\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle = (\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \downarrow\uparrow\uparrow)/\sqrt{3}$$

$$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle = (\downarrow\downarrow\downarrow)$$

- Completely symmetric

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{1,2} = (\uparrow\downarrow - \downarrow\uparrow)\uparrow/\sqrt{2}$$

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle_{1,2} = (\uparrow\downarrow - \downarrow\uparrow)\downarrow/\sqrt{2}$$

Or

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{2,3} = \uparrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$$

$$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle_{2,3} = \downarrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$$

- Partially antisymmetric
- Can also have spin ½ antisymmetric in 1 & 3 but this not independent as $\Psi_{13} = \Psi_{12} + \Psi_{23}$

*Baryon **Decuplet** Spin-Flavor Wavefunction*

- Color wavefunction is always antisymmetric for baryons.
- $\mathbf{\Psi_s}$(spin) is **symmetric** for decuplets implies that $\mathbf{\Psi_s}$(flavor) must also be **symmetric**
- Therefore

$$\mathbf{\Psi}(\text{baryon decuplet}) = \mathbf{\Psi_s}(\text{spin})\ \mathbf{\Psi_s}(\text{flavor})$$

For example, consider the case of $\mathbf{\Delta^+}$

$$\left|\Delta^+ : \frac{3}{2}\ -\frac{1}{2}\right\rangle = \left[\frac{1}{\sqrt{3}}(uud + udu + duu)\right]\left[\frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow)\right]$$

$$= \frac{1}{3}\begin{bmatrix} u(\downarrow)u(\downarrow)d(\uparrow) + u(\downarrow)u(\uparrow)d(\downarrow) + u(\uparrow)u(\downarrow)d(\downarrow) \\ + u(\downarrow)d(\uparrow)u(\downarrow) + u(\downarrow)d(\uparrow)u(\downarrow) + u(\downarrow)d(\uparrow)u(\downarrow) \\ + d(\downarrow)u(\uparrow)u(\downarrow) + d(\downarrow)u(\uparrow)u(\downarrow) + d(\downarrow)u(\uparrow)u(\downarrow) \end{bmatrix}$$

*Baryon **Octet** Spin-Flavor Wavefunction*

- States of **mixed symmetry / partially antisymmetric** are put together to make **a completely symmetric** combination.
- **Product** of two antisymmetric function is itself symmetric.
- **Ψ_{12}(spin)Ψ_{12}(flavor)** is symmetric in 1 & 2; **Ψ_{23}(spin)Ψ_{23}(flavor)** in 2&3 and **Ψ_{13}(spin)Ψ_{13}(flavor)** in 1&3 .
- Thus

$$\Psi(\text{baryon Octet}) = \{2^{1/2}/3\}[\Psi_{12}(\text{spin})\,\Psi_{12}(\text{flavor}) + \Psi_{23}(\text{spin})\,\Psi_{23}(\text{flavor}) + \Psi_{13}(\text{spin})\,\Psi_{13}(\text{flavor})]$$

For example, consider the case of **Proton (uud),** using spin combination for octets

$$|p\rangle = \left|\frac{1}{2}\;\frac{1}{2}\right\rangle$$

→ 2 quarks in up spin(+ ½) + 1 down spin (- ½) → total spin ½ & projection is +½

$$\left|p:\frac{1}{2}\;\frac{1}{2}\right\rangle = \frac{\sqrt{2}}{3}\left[\frac{1}{2}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)(udu - duu) + \frac{1}{2}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(uud - udu) + \frac{1}{2}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(uud - duu)\right]$$

$$= \frac{1}{\sqrt{18}}\left[uud\left(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right) + udu\left(2\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow\right) + ddu\left(2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow\right)\right]$$

$$= \frac{2}{3\sqrt{2}}[u(\uparrow)u(\uparrow)d(\downarrow)] - \frac{1}{3\sqrt{2}}[u(\uparrow)u(\downarrow)d(\uparrow)] - \frac{1}{3\sqrt{2}}[u(\downarrow)u(\uparrow)d(\uparrow)] + permutations$$

Bibliography

Introduction to Elementary Particles

by Griffiths D.