## Fraunhofer Diffraction due to two slits (Double slits):

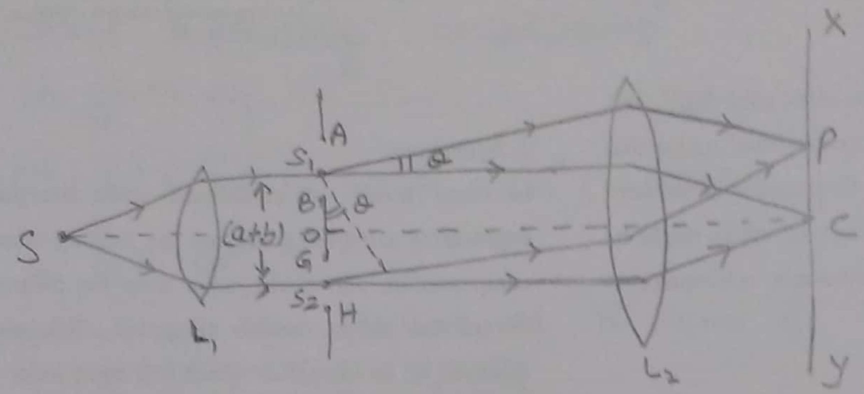



fig.(1)

Let AB & GH be two parallel slits of equal width 'a' & separated by an opaque distance 'b' as shown in fig.(1). A plane wavefront is normally incident on the slits. In this case both the phenomenon of Interference & Diffraction will take place.

When the wavefront strikes the slits then all the points within the slits become the sources of secondary wavelets travelling in all directions. The waves travelling in the direction of incident ray focus at 'c' & those travelling in a direction making an angle 'θ' focus at 'P'.

So the resultant amplitude 'R' due to all the waves diffracted through angle 'θ' is;

$$R = \frac{A\sin\alpha}{\alpha} \quad -(1) \qquad \text{where } \alpha = \frac{\pi a\sin\theta}{\lambda}$$

For simplicity, we may consider the resultant of all the waves a single wave of amplitude 'R' & phase 'α' starting from the middle points of $S_1$ & $S_2$ of each slit to direction 'θ'.

The phase difference b/w the waves starting from extreme points of a slit is

$$2\cdot\frac{\pi a\sin\theta}{\lambda} = 2\alpha$$

So the resultant amplitude at 'P' will be due to the interference b/w the two waves of same amplitude 'R' & phase diff. 'φ'.

From fig.(1), the path diff. b/w the waves reaching 'P' from $S_1$ & $S_2$ is: $S_2M = (a+b)\sin\theta$

so the phase diff. is : $\phi = \frac{2\pi}{\lambda}\cdot(a+b)\sin\theta$

From the triangle of amplitudes (fig.(2)), the resultant ~~intensity~~ amplitude at 'p' is given by;

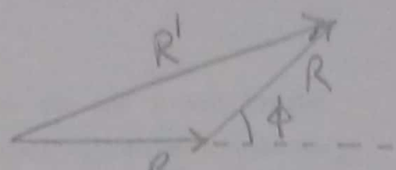



fig (2)

$$R'^2 = R^2 + R^2 + 2R.R.\cos\phi = 2R^2(1+\cos\phi)$$

$$\Rightarrow R'^2 = 4R^2 \cos^2\frac{\phi}{2}$$

On substituting the values of 'R' and 'φ' in above eqⁿ: we have

$$\boxed{R'^2 = 4A^2 \frac{\sin^2\alpha}{\alpha^2} . \cos^2\beta} \quad \text{—— (2)} \quad *$$

where $\beta = \frac{\phi}{2} = \frac{\pi}{\lambda}(a+b).\sin\theta$ —— (3) *

so Intensity at 'p' in the diffraction pattern is:

$$\boxed{I = 4A^2 . \frac{\sin^2\alpha}{\alpha^2} . \cos^2\beta} \quad \text{—— (4)} \quad *$$

so resultant intensity [eqⁿ (4)] has 2 factors;

(i) The factor $A^2\frac{\sin^2\alpha}{\alpha^2}$, represents the intensity distⁿ in the diffraction pattern due to any individual slit.

(ii) The factor $\cos^2\beta$ gives the interference pattern due to waves starting from two parallel slits.

* So the resultant intensity at any point on the screen is given by the product of these two factors & will be zero when either of these factors is zero.

(i) The diffraction term $A^2.\frac{\sin^2\alpha}{\alpha^2}$ gives the central maxima in the direction $\theta = 0$ having alternate minima & scondary maxima of decreasing intensity on either side. fig 3(c)

The angular positions of minima are given by;

* $\boxed{\sin\alpha = 0}$; $\alpha \neq 0$ so $\boxed{\alpha = \pm m\pi}$ ; $m = 1, 2, 3 \ldots$

OR $\frac{\pi a \sin\theta}{\lambda} = \pm m\pi \Rightarrow \boxed{\sin\theta = \pm\frac{m\lambda}{a}}$ ; $m = 1, 2, \ldots$

The position of secondary minima approach to

* $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$

(ii) According to the second factor $\cos^2\beta$, the intensity will be maximum when

* $\boxed{\cos^2\beta = 1}$ i.e. $\beta = \pm n\pi$ ; $n = 0, 1, 2, \ldots$

or $\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi \Rightarrow \boxed{(a+b)\sin\theta = \pm n\lambda}$ *

i.e. when $n=0$, $\theta = 0$ so the central maxima of interference pattern lies along the direction of incident light, which is called principal maxima of zero order.

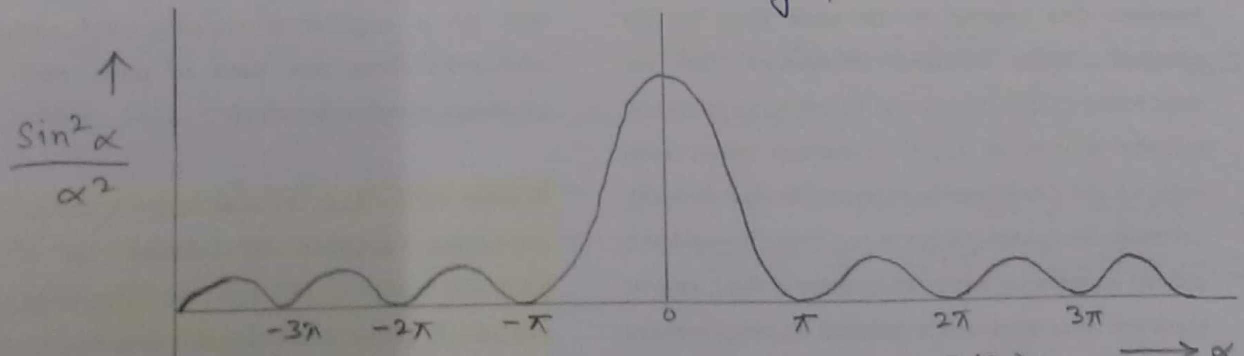
The central maxima of diffraction pattern also lies along this direction. So at this point the waves arrive in the same phase. Hence the intensity of the central maxima is maximum.

Now the intensity will be minimum when

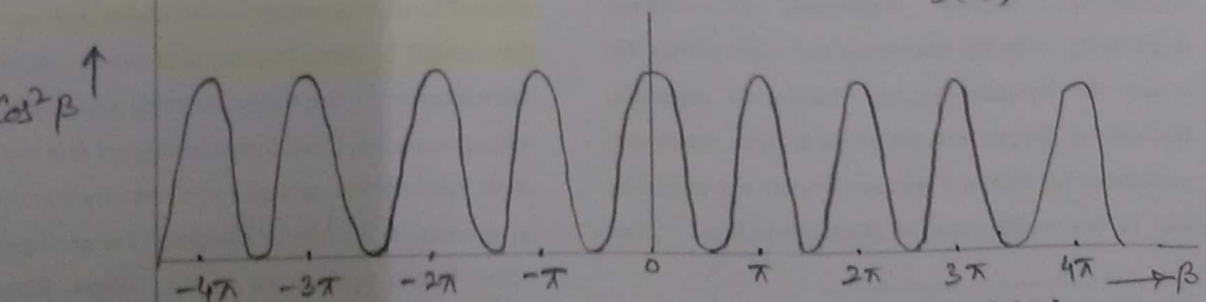
* $\boxed{\cos^2\beta = 0}$ i.e. $\beta = \pm(2n+1)\frac{\pi}{2}$ ; $n = 0, 1, 2, \ldots$

or $\boxed{(a+b)\sin\theta = \pm(2n+1)\lambda/2}$ *

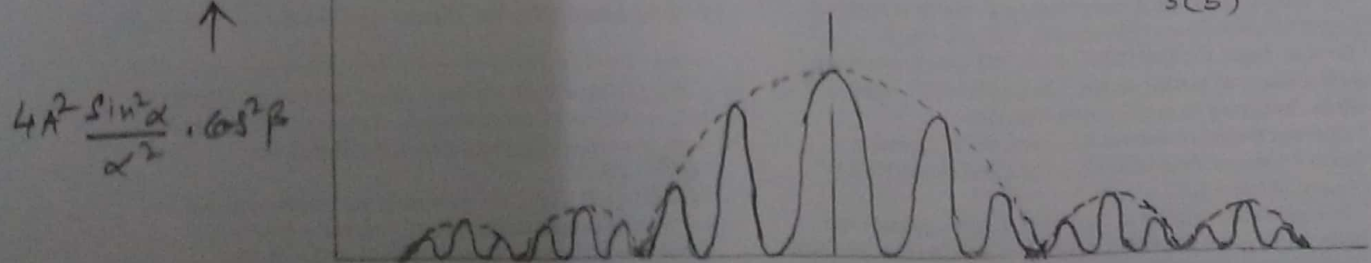
figs 3(a) & 3(b) represent the intensity distn (pattern) determined by the factors $\frac{A^2\sin^2\alpha}{\alpha^2}$ & $\cos^2\beta$ resp. fig 3(c) is the resultant curve of 3(a) & 3(b). It is obtained by multiplying the ordinates of first two curves at every point. *




3(a)




3(b)




3(c)

## Missing Orders in a Double slit diffraction pattern :-

The directions of interference maxima is given by:

$$(a+b)\sin\theta = \pm n\lambda \quad \text{——— (1)}$$

The directions of diffraction minima is given by:

$$a\sin\theta = \pm m\lambda \quad \text{——— (2)}$$

Now let 'a' is constant and 'b' varies.

from eq^ns (1) & (2): $\frac{a+b}{a} = \frac{n}{m}$ ——— (3)

(i) If $a=b$ then $\frac{n}{m} = 2$ or $n = 2m = 2, 4, 6, \ldots$ (as $m = 1, 2, 3 \ldots$)

So 2nd, 4th, 6th ... order interference maxima will be absent or missing in the diffraction pattern.

when $m=1$; $n=2$ so there will be three interference maxima ($n = 0, \pm 1$) in the central diffraction maximum.

(ii) If $2a = b$ then from eq^n (3) $\frac{n}{m} = 3$ or $n = 3m = 3, 6, 9 \ldots$ (as $m = 1, 2 \ldots$)

So 3rd, 6th, 9th ... order interference maxima will be absent. and there will be 5 interference maxima in the central diffraction maximum (i.e. $n = 0, \pm 1, \pm 2$)

(iii) If $a+b = a$ i.e. if $b=0$ then two slits join & all the orders of interference maxima will be absent.

* So we see when 'b' increases, the number of interference maxima in the region of central diffraction maximum increases.

Class - B.Sc. II^nd sem. Section - B

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