INDEX NUMBERS

Index numbers are indicators which reflect the relative changes in the level of a phenomenon in any given period called the current period with respect to its value in any some base period i.e. fixed selected for comparison. The variable under consideration may be that price of particular commodities like steel, gold etc.,volume of trade,inflation, factory production,imports and exports, national income, cost of living of persons of a particular commodity and so on.

"Index numbers are used to measure the changes in some quantity which we cannot observe directly".

For example, we are not able to directly measure changes in a country's business activity. However, we can study relative changes in business activity by examining the variations in the values of some of the factors that influence business activity, and which can be measured directly.

DEFINITIONS OF INDEX NUMBERS

Some of the important definitions of Index Numbers are:

According to **Croxton** and **Cowden** *"index numbers are devices for measuring differences in the magnitude of a group of related variables".*

According to **Patterson** *"index number is a ratio of two numbers expressed as a percent."*

Spiegel defined index numbers as "a statistical measure designed to show changes in variable or a group of related variables with respect to time, geographic location or other characteristics such as income, profession etc."

An index number is a specialized average designed to measure the change in a group of related variables over a period of time.

For example, if the index number of industrial production (IIP) in 2018 is 106 as compared to Industrial production in 2010, it means that there is an increase of 6 per cent in the industrial production in 2018 as compared to 2010. Index number relates to a variable(s) in a given period to the same variable(s) in another period. The other period is known as base period. Thus, in index numbers current period is compared to based period to know the percentage change in current period as compared to the based period.

If the index number pertains to a single variable, it is known as univariate index and if it relates to a group of variables it is considered as a composite index.

TWO MAIN TYPES OF INDEX NUMBERS

- **Price Index Number:** this measures the relative changes in prices between two specific points in time.
- **Quantity Index Number:** considered to measure changes in the physical quantity of goods produced, purchased or sold of one item or group of items.

USES OF INDEX NUMBERS

For business and economic analysis index numbers are essential tools. The utility of index numbers as follows:

1. **Index numbers as economic barometer:** Index numbers are indispensable tools for the management personal of any government organization or individual business concern and in business planning and formulation of executive decisions
2. **Index numbers are helpful in framing suitable policies:** Many economic and business policies are based on the information provided by index number. For example, Dearness Allowance is fixed by considering the cost of living index number
3. **Index numbers measure the purchasing power of money:** There exists an inverse relation between the value of money and the price level. Cost of living index determines whether the real wages are falling or rising,money wages remaining unchanged
4. **Index numbers are used for deflation:** Consumer Price indices or cost of living index are used for deflation of net national product,income value series in national accounts. The Purchasing power of money goes on changing with the change in price level. Index numbers are used to adjust the original data for price changes and thus convert nominal wages into real wages or nominal income into real income.
5. **Index numbers help in studying trends and tendencies:** Since Index numbers study the relative changes in the level of any phenomenon over a period of time, they can be used to study the trend of the phenomenon in a time series data..

STEPS OR PROBLEMS IN THE CONSTRUCTION OF PRICE INDEX NUMBERS

The construction of the price index numbers involves the following steps or problems:

1. The Purpose of Index Numbers

The first and foremost task before constructing any index number is to define in clear and in concrete terms the objective or purpose of index numbers. The knowledge about purpose of index number helps us to collect relevant data, select appropriate commodities, assign suitable weights and use proper techniques. Failure to decide clearly the purpose of the index would lead to confusion and wastage of time with no fruitful results .

2. Selection of Base Year

The first step or the problem in preparing the index numbers is the selection of the base year. The base year is defined as that year with reference to which the price changes in other years are compared and expressed as percentages. The following points in conformity with the objectives of index should serve as a guidelines for selecting a base period:

(i) The base year should be a normal year. In other words, it should be free from abnormal conditions like wars, famines, floods, political instability, etc.

(ii) Base year should not be too distant from the given period. Due to the rapid and dynamic pace of events these days, it is desirable that the base periods should not be very far off from the current period .if the time lag are too much between the current and the base periods then there may be change in tastes, customs, habits and fashions of the people thereby affecting the consumption pattern of the various commodities to a marked extent.

(iii) Base year can be selected in two ways– (a) through fixed base method in which the base year remains fixed; and (b) through chain base method in which the base year goes on changing, e.g., for 2018 the base year will be 2017, for 2017 it will be 2016, and so on.

3. Selection of Commodities

The commodities selected should be relevant for the purpose of index numbers. Since all commodities cannot be included, only representative commodities should be selected keeping in view the purpose and type of the index number.

In selecting items, the following points are to be kept in mind:

(a) The items should be representative of the tastes, habits and customs of the people.

(b) Items should be recognizable,

(c) Items should be stable in quality over two different periods and places.

(d) The economic and social importance of various items should be considered

(e) The items should be fairly large in number.

(f) Classifying the whole relevant groups of items or commodities into relevant homogenous groups such as: Food (cereals,rice, wheat,pulses,grams etc., milk and milk products,fruits , vegetables,meat ,poultry items etc.,clothing,fuel and lighting,house rent and so on.

(g) All those varieties of a commodity which are in common use and are stable in character should be included.

4. Selecting Price Quotations

After selecting the commodities, the next problem is regarding the collection of their prices:

(a) From where the prices to be collected;

(b) Whether to choose wholesale prices or retail prices;

(c) Whether to include taxes in the prices or not etc.

While collecting prices, the following points are to be noted:

(a) Prices are to be collected from those places where a particular commodity is traded in large quantities.

(b) In selecting individuals and institutions who would supply price quotations, care should be taken that they are not biased.

(c) Selection of wholesale or retail prices depends upon the type of index number to be prepared. Wholesale prices are used in the construction of general price index and retail prices are used in the construction of cost-of-living index number.

5. Selection of Average

For the construction of index numbers ,usually the choice lies between arithmetic mean ,geometric mean and median. Theoretically, geometric mean is the most appropriate average for this purpose:

(i) In index numbers,we deal with ratios and relative changes and geometric mean gives equal weights to equal ratios of change.

(ii) It gives more importance to small items and less importance to large items and is therefore not duly affected by extreme items.

6. Selection of Weights

Generally, all the commodities included in the construction' of index numbers are not of equal importance. But in general ,we follow two types of index numbers:

(i) Unweighted index numbers: The index numbers constructed without assigning any weights to different items are called unweighted index numbers.

(ii) Weighted Index Numbers: These are obtained afyer assigning weights to different items according to their relative importance in the group. Therefore, if the index numbers are to be representative, proper weights should be assigned to the commodities according to their relative importance.

For example, the prices of books will be given more weightage while preparing the cost-of-living index for teachers than while preparing the cost-of-living index for the workers. Weights should be unbiased and be rationally and not arbitrarily selected.

7. Selection Appropriate Formula

The selection of a suitable method for the construction of index numbers is the final step. This choice first depends on data available. An index number should be such which satisfies both Factor reversal test and time reversal test.

METHODS OF CONSTRUCTING THE INDEX NUMBERS

The various methods used for the construction of index numbers are :

1. Unweighted Methods

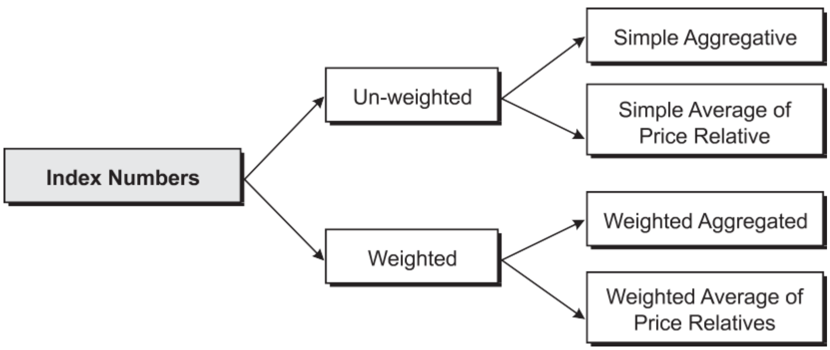
(a) Simple Aggregative Method

(b) Simple average of relatives(price relative)method

2. Weighted Methods

(a) Weighted Aggregative Method

(b) Weighted average of relative method.



The choice of method depends upon the availability of data, degree of accuracy required and the purpose of the study.

Construction of price index numbers through various methods can be understood with the help of the following examples:

1. Simple Aggregative Method

In this method, the index number is equal to the sum of prices for the year for which index number is to be found divided by the sum of actual prices for the base year.

The formula for finding the index number through this method is as follows:

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

Where,

P_{01} Stands for the index number

ΣP_1 Stands for the sum of the prices for the year for which index number is to be found.

ΣP_0 Stands for the sum of prices for the base year.

Commodity	Prices in Base Year 2017 (in `) P_0	Prices in Current Year 2018 (in `) P_1
A	10	20
B	15	25
C	40	60
D	25	40
Total	$\Sigma P_0 = 90$	$\Sigma P_1 = 145$

Index Number $(P_{01}) = \dfrac{\Sigma P_1}{\Sigma P_0} \times 100$; $P_{01} = \dfrac{145}{90} \times 100$; $P_{01} = 161.11$

2. Simple Average of Price Relatives Method

In this method, the index number is equal to the sum of price relatives divided by the number of items and is calculated by using the following formula:

$$P_{01} = \frac{\Sigma R}{N}$$

Where, SR stands for the sum of price relatives i.e., $R = \dfrac{P_1}{P_0} \times 100$ and

N stands for the number of items.

Example:

Commodity P_0	Base Year Price (in `) P_1	Current Year Price (in `)	Price Relatives $R = \frac{P_1}{P_0} \times 100$
A	10	20	$\frac{20}{10} \times 100 = 200.0$
B	15	25	$\frac{25}{15} \times 100 = 166.7$
C	40	60	$\frac{60}{40} \times 100 = 150.0$
D	25	40	$\frac{40}{25} \times 100 = 160.0$
N = 4			$\Sigma R = 676.7$

$$\text{Index Number } (P_{01}) = \frac{\Sigma R}{N}$$

$$P_{01} = \frac{676.7}{4}$$

$$P_{01} = 169.2$$

3. Weighted Aggregative Method

In this method, appropriate weights are assigned to various commodities to reflect their relative importance in the group. For construction of price index numbers, quantity weights are used. Many formulae have been developed to estimate index numbers on the basis of quantity weights.

Some of them are explained below:

(i) **Laspeyre's Formula:** In this formula, the quantities of base year are accepted as weights.

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$$

Where P_1 is the price in the current year; P_0 is the price in the base year; and q_0 is the quantity in the base year.

(ii) **Paasche's Formula:** In this formula, the quantities of the current year are accepted as weights.

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Where q_1 is the quantity in the current year.

(iii) **Dorbish and Bowley's Formula:** Dorbish and Bowley's formula for estimating weighted index number is as follows:

$$P_{01} = \frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100 \quad \text{or } P_{01} = \frac{L + P}{2}$$

Where L is Laspeyre's index and P is Passche's Index.

(iv) **Fisher's Ideal Formula:** In this formula, the geometric mean of two indices (i.e. Laspeyre's Index and Paasche's Index) is taken:

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100 \quad \text{or } P_{01} = \sqrt{L \times P} \times 100$$

Where L is Lespeyre's Index and P is Paasche's Index.

Example:

Commodity	Base Year		Current Year		p_0q_0	p_1q_0	p_0q_1	p_1q_1
	p_0	q_0	p_1	q_1				
A	10	5	20	2	50	100	20	40
B	15	4	25	8	60	100	120	200
C	40	2	60	6	80	120	240	360
D	25	3	40	4	75	120	100	160
Total					265 Σp_0q_0	440 Σp_1q_0	480 Σp_0q_1	760 Σp_1q_1

(i) Laspeyre's Formula:

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$$

$$P_{01} = \frac{440}{265} \times 100 = 166.04$$

(ii) Paasche's Formula:

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$$P_{01} = \frac{700}{480} \times 100 = 158.3$$

(iii) Dorbish and Bowley's Formula:

$$P_{01} = \frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100 = 162.2$$

$$P_{01} = \frac{\frac{440}{265} + \frac{760}{480}}{2} \times 100 = 162$$

(iv) Fisher's Ideal Formula:

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

$$P_{01} = \sqrt{\frac{440}{265} \times \frac{760}{480}} \times 100 = 162.1$$

(v) **Marshall and Edgeworth Method:** The formula given by Marshall and Edgeworth for constructing an index number is known as Marshall Edgeworth's method. In this method, both the current year and base year quantities are considered.

$$P_{01} = \frac{\Sigma\,[\,p_1(q_0 + q_1)/\,2]}{\Sigma\,[\,p_0\,(q_0 + q_1)/\,2]} \times 100$$

$$= \frac{\Sigma\,[\,p_1(q_0 + q_1)]}{\Sigma\,[\,p_0\,(q_0 + q_1)]} \times 100$$

$$= \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

Thus, the Marshall Edgeworth's index number is given by–

$$P_{01}(ME) = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

(vi) **Kelly's Method:** Kelly's index is a weighted aggregative price index which uses fixed weights. Truman L. Kelly has suggested the following formula for constructing index number:

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$$

Here weights are the quantities which may refer to some period, not necessarily the base year or current year. Thus the average quantity of two or more years may be used as weights. If in the Kelly's formula the average of the quantities of two years is used as weights, the formula becomes:

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$$

where, $q = \frac{q_0 + q_1}{2}$

4. Weighted Average of Relatives Method

In this method also different weights are used for the items according to their relative importance.

The price index number is found out with the help of the following formula:

Weighted Index number of the current year = $\frac{\Sigma IV}{\Sigma V}$

where I stands for Price Relatives of the current year and V stands for the values of the base year.

$$V = p_0 q_0$$

Illustration: From the data given below, calculate the Weighted Index Number by using weighted average of Relatives.

Commodities	Units	Base Yr. Qty.	Base Year's Price	Current Yr. Price
A	Quintal	7	16	19.6
B	K.g.	6	2	3.2
C	Dozen	16	5.6	7.0
D	Metre	21	1.5	1.4

Solution:

The Price relative of the current year = $\frac{\text{Current Year's Price}}{\text{Base Year's Price}} \times 100$

The value of the base year = Quantity of base year × Price of the base year

Commodities	Price Relatives $\left(I = \frac{P_t}{P_a} \times 100 \div\right)$	Value of Weights i.e. $V = p_0 q_0$	Weights × Price Relatives $V \times t$
A	122.5	112.0	13,720
B	160.0	12.0	1,920
C	125	89.6	11,200
D	93.3	31.5	2,939
		$\Sigma V = 245.1$	$\Sigma IV = 29,779$

$$\text{Weighted Index Number of the Current Year} = \frac{\Sigma IV}{\Sigma V}$$

$$= \frac{29,779}{245} = 121.5$$

TESTS OF ADEQUACY OF INDEX NUMBER FORMULAS

(1) Unit Test
(2) Time Reversal Test
(3) Factor Reversal Test
(4) Circular Test

(1) Unit Test

The unit test requires that the formula constructing an index should be independent of the units in which, or for which, prices and quantities are quoted.

(2) Time Reversal Test

Time reversal test is a test to determine whether a given method will work both ways in time, forward and backward. In the words of Fisher, "The test is that the formula for calculating the index number should be such that it will give the same ratio between one point of comparison and the other, no matter which of the two is taken as base." In other words, when the index numbers of the two years are constructed by reversing the base year ,they should be reciprocals of each other so that their product is unity.. Symbolically, the following relation should be satisfied:

$$P_{01} \times P_{10} = 1$$

Where P_{01} is the index for time "I" on time "0" as base and P_{10} is the index for time "0" on time "I" as base. If the product is not unity, there is said to be a time bias in the method.

The test is not satisfied by Laspeyres method and the Paasche method as can be seen below:

When Laspeyres method is used:

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}; \quad P_{10} = \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1};$$

$$P_{01} \times P_{10} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \neq 1$$

and the test is not satisfied.

When Paasche method is used:

$$P01 = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}; \quad P_{10} = \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1};$$

$$P_{01} \times P_{10} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \neq 1$$

and the test is not satisfied.

There are five methods which do satisfy the test:

1) The Fisher's ideal formula,
2) Simple geometric mean of price relatives,
3) Aggregates with fixed weights,
4) The weighted geometric mean of price relatives if we use fixed weights, and
5) Marshall-Edgeworth method.

Let us now see how Fisher's Ideal formula satisfies the test.

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

Changing time, i.e., 0 to 1 and 1 to 0.

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}} = \sqrt{1} = 1$$

Since $P_{01} \times P_{10} = 1$, the Fisher's ideal index satisfies the test.

(3) Factor Reversal Test

Another test suggested by Fisher is known as factor reversal test. It holds that the product of a price index and the quantity index should be equal to the corresponding value index. In the words of Fisher, "Just as each formula should permit the interchange the prices and quantities without giving inconsistent results, so it ought to permit the interchange of the two times without giving inconsistent results, i.e., the two results multiplied together should give the true value ratio".

The test says that change in price multiplied by change in quantity should be equal to total change in value. If $\mathbf{P_{01}}$ is a price index for the current year with reference to base year and $\mathbf{Q_{01}}$ is the quantity index for the current year.

Then,
$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

This test is satisfied only by Fisher's ideal index method.

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

Changing *p* to *q* and *q* to *p*.

$$Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$\therefore \quad P_{01} \times Q_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} = \sqrt{\frac{(\Sigma q_1 p_1)^2}{(\Sigma p_0 q_0)^2}} = \frac{(\Sigma p_1 q_1)}{(\Sigma p_0 q_0)}$$

(4) Circular Test

According to this, if indices are constructed for year ,one based on year zero,for year two based on year one and for year zero based on year two, the product of all the indices should be equal to 1.

Symbolically,

$$P_{01} \times P_{12} \times P_{23} \ldots P_{no} = 1$$

This test is satisfied by (i) Simple aggregative method and (ii) Kelley's method.

Why is Fisher's Index said to be ideal?

Fisher's index satisfies both time reversal test and factor reversal test and also takes into account both base year and current year quantities. Therefore,it is called as ideal.

CHAIN BASE INDEX NUMBERS

Chain base index number is one in which the figures for each year are first expressed as percentages of the preceeding year. These are known as Link Relatives. We then need to chain them together by successive multiplication to form a chain index.

Thus, unlike fixed base methods, in this method, the base year changes every year. Hence, for the year 2001, it will be 2000, for 2002 it will be 2001, and so on.

Steps in the Construction of Chain Index Numbers

1. Calculate the link relatives by expressing the figures as the percentage of the preceding year. Thus,

$$\text{Link Relatives of current year} = \frac{\text{Price of current year}}{\text{Price of previous year}} \times 100$$

2. Calculate the chain index by applying the following formula:

$$\text{Chain Base Index (CBI)} = \frac{\text{Current year LR} \times \text{Preceding year chain index}}{100}$$

3. The chain base index can be converted into fixed base index by this formula:

$$\text{Fixed Base Index (FBI)} = \frac{\text{Current year CBI} \times \text{Previous year FBI}}{100}$$

Advantages of Chain Index Numbers Method

1. This method allows the addition or introduction of the new items in the series and also the deletion of obsolete items.
2. It is free from seasonal variations.
3. It permits the adjustment of weights as frequently as possible.
4. It is very useful in economic and business data where comparisons are to be made with the previous period.

5. In an organization, management usually compares the current period with the period immediately preceding it rather than any other period in the past. In this method, the base year changes every year and thus it becomes more useful to the management.

Disadvantages of Chain Index Numbers Method

1. It is complicated and time consuming.
2. Link relatives are percentage of previous year figures. The long range comparisons of chained percentages are not strictly valid.
3. Under this method, if the data for any one of the year is not available then we cannot compute the chain index number for the subsequent period. This is so because we need to calculate the link relatives, which are not possible to be calculated in this case.
4. In case an error occurs in the calculation of any of the link relatives, then that error gets compounded and all the subsequent link relatives will also become incorrect. Thus, the entire series will give a misrepresented picture.

Difference between Fixed Base index and Chain Base Index

In fixed base index, indices for various years are constructed by taking the value of a particular year as the base. The base year remains the same throughout the series of the index.

In chain base index ,the figures for each year are expressed as percentage of the preceding year. These percentage are then chained together by successive multiplication to form a series of chain indices. In chain base index ,the base year changes from year to year.

Conversion of Chain base Index number to Fixed base Index number

$$\text{Current year FBI} = \frac{\text{Current year CBI} \times \text{Previous year FBI}}{100}$$

The fixed base index for the first year will be taken the same as the chain base index

BASE SHIFTING, SPLICING AND DEFLATING OF INDEX NUMBERS

Base shifting is a technique of changing the old base period to the new base period .The reasons for base shifting are:

(i) To state the series in terms of a more recent period.

(ii) To compare various computed on different base periods.

This can be done using the following relation as:

$$\text{Shifted price index} = \frac{\text{Original price index}}{\text{Price index for new base year}} \times 100$$

The following are the index numbers of a commodity taking 2011 as the base.

Year	2011	2012	2013	2014	2015
Index Numbers	100	120	150	180	225

Find the index numbers by changing the base to 2014.

Solution:

Year	Old Index Numbers (Base Year 2011)	New Index Numbers (Base Year 2014)
2011	100	$\frac{\text{Original price index}}{\text{Price index for 1994}} \times 100 = \frac{100}{180} \times 100 = 55.56$
2012	120	$\frac{120}{180} \times 100 = 66.67$
2013	150	$\frac{150}{180} \times 100 = 83.33$
2014	180	$\frac{180}{180} \times 100 = 100$
2015	225	$\frac{225}{180} \times 100 = 125$

Splicing

Sometimes,the construction of an index number series is discontinued for the reason of its base becoming too old. A new set of index numbers may be computed with some recent years as the base. It may be desired to set the new indices with the old ones. The statistical procedure which connects an old index number series is called splicing.

Year	Old Series (Base Year 1998)	New Series (Base Year 2001)	Splicing (Base Year 1998)
1998	100		100
1999	140		140
2000	170		170
2001	200	100	$\frac{100 \times 200}{100} = 200$
2002		110	$\frac{110 \times 200}{100} = 220$
2003		130	$\frac{130 \times 200}{100} = 260$

Deflating

Year	Wages (`)	Price Index	Real Wages (`)	Index of Real Wages
2003	2000	100	$\frac{2000}{100} \times 100 = 2000$	100.00
2004	2400	160	$\frac{2400}{160} \times 100 = 1500$	75.00

Year	Wages (`)	Price Index	Real Wages (`)	Index of Real Wages
2005	3500	280	$\frac{3500}{280} \times 100 = 1250$	62.50
2006	3600	300	$\frac{3600}{300} \times 100 = 1200$	60.00

EXERCISE

1. What is factor reversal test? Show that Fisher's index satisfied this test.
2. 'An Index number is a special type of average'. Elaborate on the statement.
   (B.Com (Hons.), L.U., 2015)
3. State the uses and significance of index numbers. ***(B.Com (Hons.), L.U., 2016)***
4. Describe the uses of index numbers. ***(B.Com (Hons.), L.U., 2018)***
5. Discuss the essential features of Index Numbers. Wheat are the difference methods used for constructing weighted index numbers? Which of them is most appropriate and why?
   (B.Com (Hons.), L.U., 2018)
6. What are the limitations of Index Numbers? ***(B.Com (Hons.), L.U., 2017)***

SOLVED EXERCISE

1. You are given the following figures.

Commodity	2006		2008	
	Price	Quantity	Price	Value
A	6	50	10	560
B	2	100	2	240
C	4	60	6	360
D	10	30	12	288
E	8	40	12	432

Construct Fisher's Ideal Index number and prove that this index number satisfies both the reversibility tests. ***(B.Com (Hon.), L.U., 2015)***

Solution:

Commodity	2006		2008					
	P_0	q_0	P_1	q_1	P_0q_0	P_1q_0	P_1q_1	P_0q_1
A	6	50	10	56	300	500	560	336
B	2	100	2	120	200	200	240	240
C	4	60	6	60	240	360	360	240
D	10	30	12	24	300	360	288	240
E	8	40	12	36	320	480	432	288
					1,360	**1,900**	**1,880**	**1,344**

Fisher's Ideal Index

$$P_{01} = \sqrt{\frac{\sum P_1q_2}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100$$

$$P_{01} = \sqrt{\frac{1,900}{1,360} \times \frac{1,880}{1,344}} \times 100$$

$$= 1.9542 \times 100$$

$$= 195.42$$

Time Reversal Test: It is satisfied when $P_{01} \times P_{10} = 1$, while applying the test, we omit the multiplication by 100.

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times \sqrt{\frac{\sum P_0q_1}{\sum P_1q_1} \times \frac{\sum P_0q_0}{\sum P_1q_0}}$$

$$= \sqrt{\frac{1,900}{1,360} \times \frac{1,880}{1,344}} \times \sqrt{\frac{1,344}{1,880} \times \frac{1,360}{1,900}}$$

$$= \sqrt{1} = 1$$

Since $P_{01} \times P_{10} = 1$. Therefore, fisher's ideal index satisfies 'Time Reversal Test'.

Factor Reversal Test:

$$P_{01} \times Q_{01} = V_{01}$$

$$P_{01} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} = \sqrt{\frac{1,900}{1,360} \times \frac{1,880}{1,344}}$$

$$Q_{01} = \sqrt{\frac{\sum q_1P_0}{\sum q_0P_0} \times \frac{\sum q_1P_1}{\sum q_0P_1}} = \sqrt{\frac{1,344}{1,360} \times \frac{1,880}{1,900}}$$

$$= \frac{1,880}{1,360} = V_{01}$$

Thus, Fisher's Index satisfies time reversal test as well as factor reversal test.

9. From the data given below, prove that the Fisher's index satisfies time reversal .

Commodity	Price in 2010	Value in 2010	Price in 2015	Value in 2015
A	5	50	6	72
B	7	84	10	80
C	10	80	12	96
D	4	20	5	30
E	8	56	8	64

Solution:

In this question, we are given price and value for each item in the base year 2010 and the current year 2015.

$\therefore$ To get the quantity, we divide the value by the price.

$$\text{Value} = \text{Price} \times \text{Quantity}$$

$$\text{Quantity} = \frac{\text{Value}}{\text{Price}}$$

Commodity	2010		2015		$q_0 = \frac{p_0q_0}{p_0}$	$q_1 = \frac{p_1q_1}{p_1}$	p_0q_1	p_1q_0
	Price	Value	Price	Value				
A	5	50	6	72	10	12	60	60
B	7	84	10	80	12	8	56	120
C	10	80	12	96	8	8	80	96
D	4	20	5	30	5	6	24	25
E	8	56	8	64	7	8	64	56
		Σp_0q_0 = 290		Σp_0q_0 = 342			Σp_0q_1 = 284	Σp_1q_0 = 357

$$P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100 = \sqrt{\frac{357}{290} \times \frac{342}{284}} \times 100$$

$$= 1.2176 \text{ (app.)} \times 100 = \mathbf{121.76 \text{ (app.)}}$$

(i) **Time Reversal Test:** $P_{01} \times P_{10} = 1$

$$P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} = \sqrt{\frac{357}{290} \times \frac{342}{284}}$$

$$P_{10} = \sqrt{\frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}} = \sqrt{\frac{284}{342} \times \frac{290}{357}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{357}{290} \times \frac{342}{284}} = \sqrt{\frac{284}{342} \times \frac{290}{357}}$$

$$= \sqrt{\frac{357}{290} \times \frac{342}{284} \times \frac{284}{342} \times \frac{290}{357}} = \sqrt{1} = 1$$

Hence, Fisher's index satisfies Time Reversal Test.

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